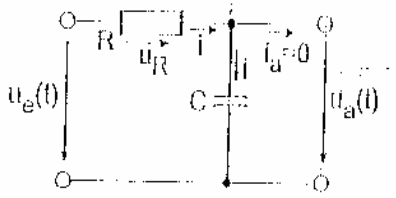
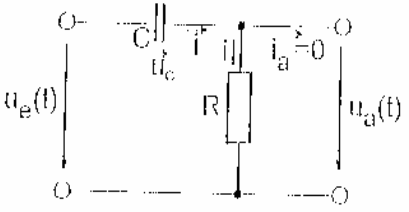
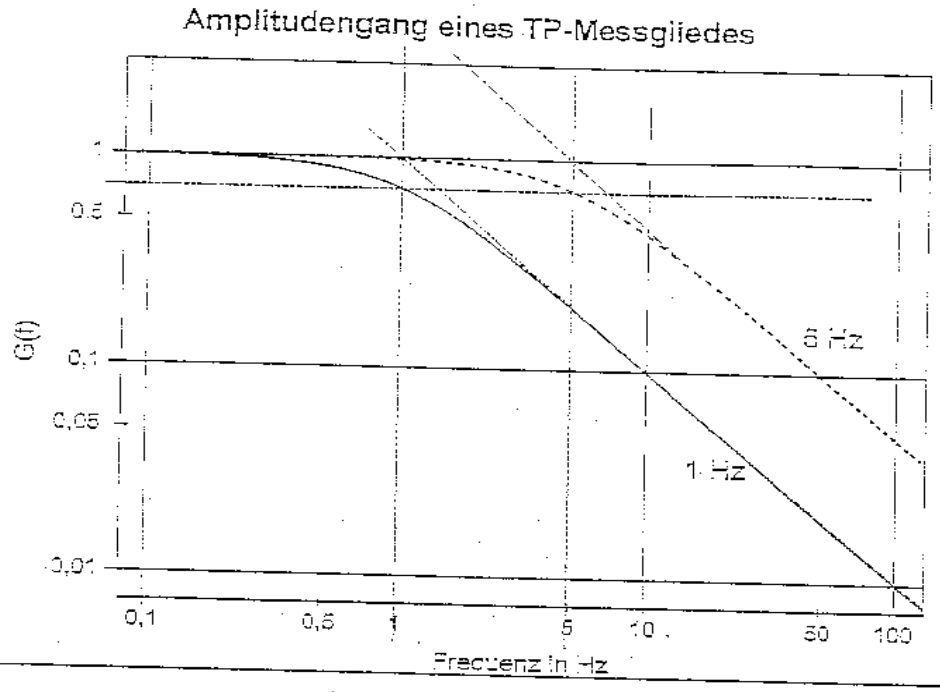
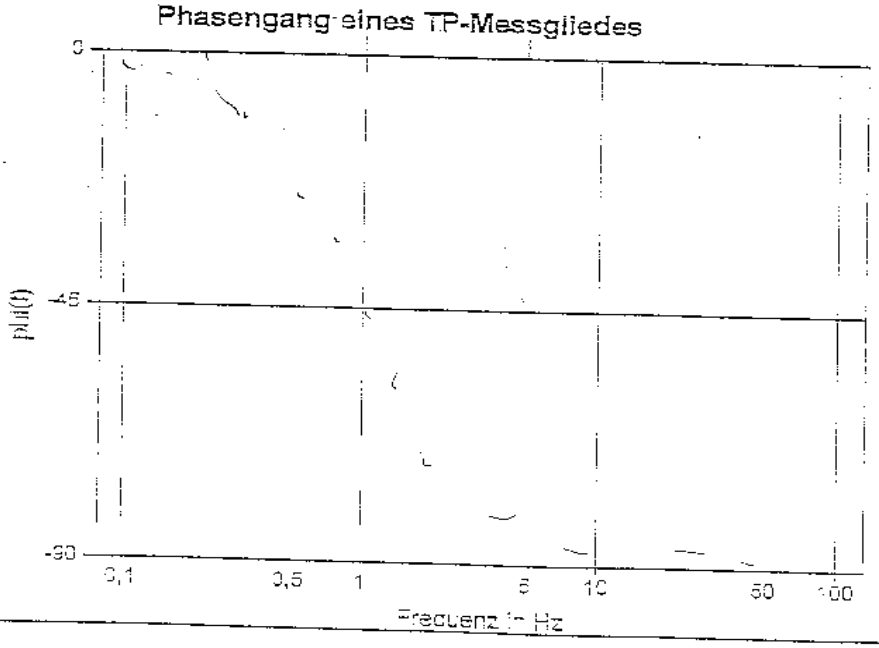
2.1 Frequenzverhalten von Sensoren

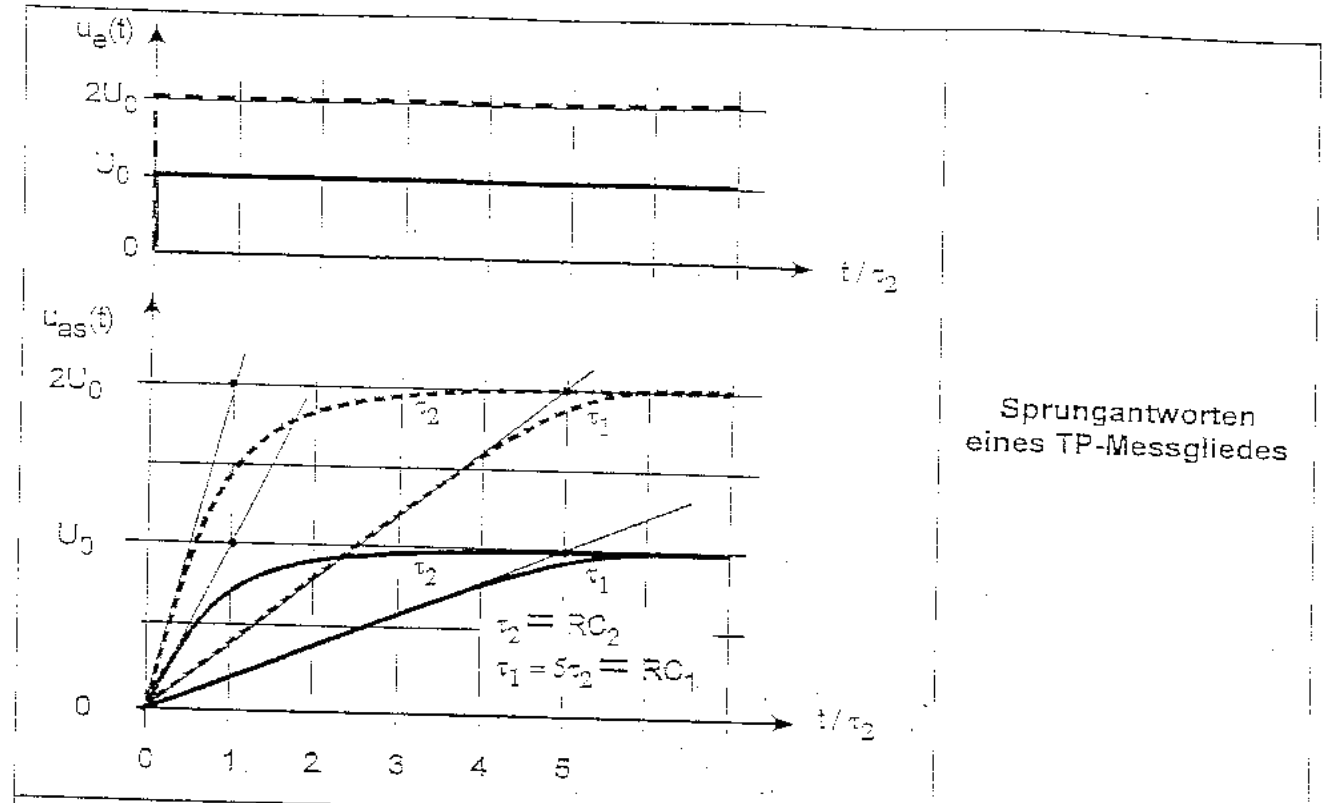
		Messglied Typ A	Messglied Typ B
a)	$\underline{G}=\frac{\underline{U}_a}{\underline{U}_e}=$	$\frac{\underline{Z}_C}{R+\underline{Z}_C}=\frac{1}{1+\frac{R}{\underline{Z}_C}}=\frac{1}{1+j\omega RC}=\frac{1}{1+j2\pi fRC}$	$\frac{R}{R+\underline{Z}_C}=\frac{1}{1+\frac{\underline{Z}_C}{R}}=\frac{j\omega RC}{1+j\omega RC}=\frac{j2\pi fRC}{1+j2\pi fRC}$
b)	$Re\{G(\omega_g)\}=Im\{G(\omega_g)\}$ $Re\{G(f_g)\}=Im\{G(f_g)\}$	$\omega_g=\frac{1}{RC}$ $f_g=\frac{1}{2\pi RC}$	$\omega_g=\frac{1}{RC}$ $f_g=\frac{1}{2\pi RC}$
c)	$\underline{G}=\frac{\underline{U}_a}{\underline{U}_e}=$	$\frac{1}{1+j\omega/\omega_g}=\frac{1}{1+jf/f_g}$	$\frac{\omega/\omega_g}{1+j\omega/\omega_g}=\frac{f/f_g}{1+jf/f_g}$
d)	$G=\lvert\underline{G}\rvert=\sqrt{Im^2+Re^2}=$	$\frac{1}{\sqrt{1+(\omega RC)^2}}=\frac{1}{\sqrt{1+\left(\frac{\omega}{\omega_g}\right)^2}}=\frac{1}{\sqrt{1+\left(\frac{f}{f_g}\right)^2}}$	$\frac{\omega RC}{\sqrt{1+(\omega RC)^2}}=\frac{\frac{\omega}{\omega_g}}{\sqrt{1+\left(\frac{\omega}{\omega_g}\right)^2}}=\frac{\frac{f}{f_g}}{\sqrt{1+\left(\frac{f}{f_g}\right)^2}}$
d)	Phasengang $\varphi=arctg\frac{\Im}{\Re}$	$arctg(\ f/f_g)$	$arctg(\frac{1}{f/f_g})$

		Messglied Typ A	Messglied Typ B
e)	$G(f_g)/G(0) =$	$1/\sqrt{2} = 0{,}707$	$1/\sqrt{2} = 0{,}707$
f)	dynamischer Fehler	in der Regel unzulässig grosse Fehler: Amplitudenfehler ca 30% Phasenfehler 45°	in der Regel unzulässig grosse Fehler: Amplitudenfehler ca 30% Phasenfehler 45°
g)	Grenzfrequenzen zahlenmässig	$0{,}16\,M\Omega;\ 1\mu F \rightarrow \tau_1 = RC_1 = 0{,}16s \Rightarrow f_{g1} = 1Hz$ $0{,}16\,M\Omega;\ 200pF \rightarrow \tau_2 = RC_2 = \frac{1}{5}0{,}16s \Rightarrow f_{g2} = 5Hz$	$0{,}16\,M\Omega;\ 1\mu F \rightarrow \tau_1 = RC_1 = 0{,}16s \Rightarrow f_{g1} = 1Hz$ $0{,}16\,M\Omega;\ 200pF \rightarrow \tau_2 = RC_2 = \frac{1}{5}0{,}16s \Rightarrow f_{g2} = 5Hz$
h)		siehe separate Diagramme	siehe separate Diagramme
i)			
i)	Schaltbild		
i)	DGL	$u_R + u_a = u_e \ ;\ i_R = i_C = i \ ;\ u_R = iR \ ;$ $u_a = u_C = \frac{1}{C}\int i\,dt \ \rightarrow i = C\dot{u}_a \ ;$ $\boxed{\Rightarrow RC\dot{u}_a + u_a = u_e}$	$u_C + u_a = u_e \ ;\ i_R = i_C = i \ ;\ u_a = iR$ $u_C = \frac{1}{C}\int i\,dt \ ; \rightarrow \frac{1}{C}\int \frac{u_a}{R}\,dt + u_a = u_e$ $\boxed{\Rightarrow RC\dot{u}_a + u_a = \dot{u}_e RC}$

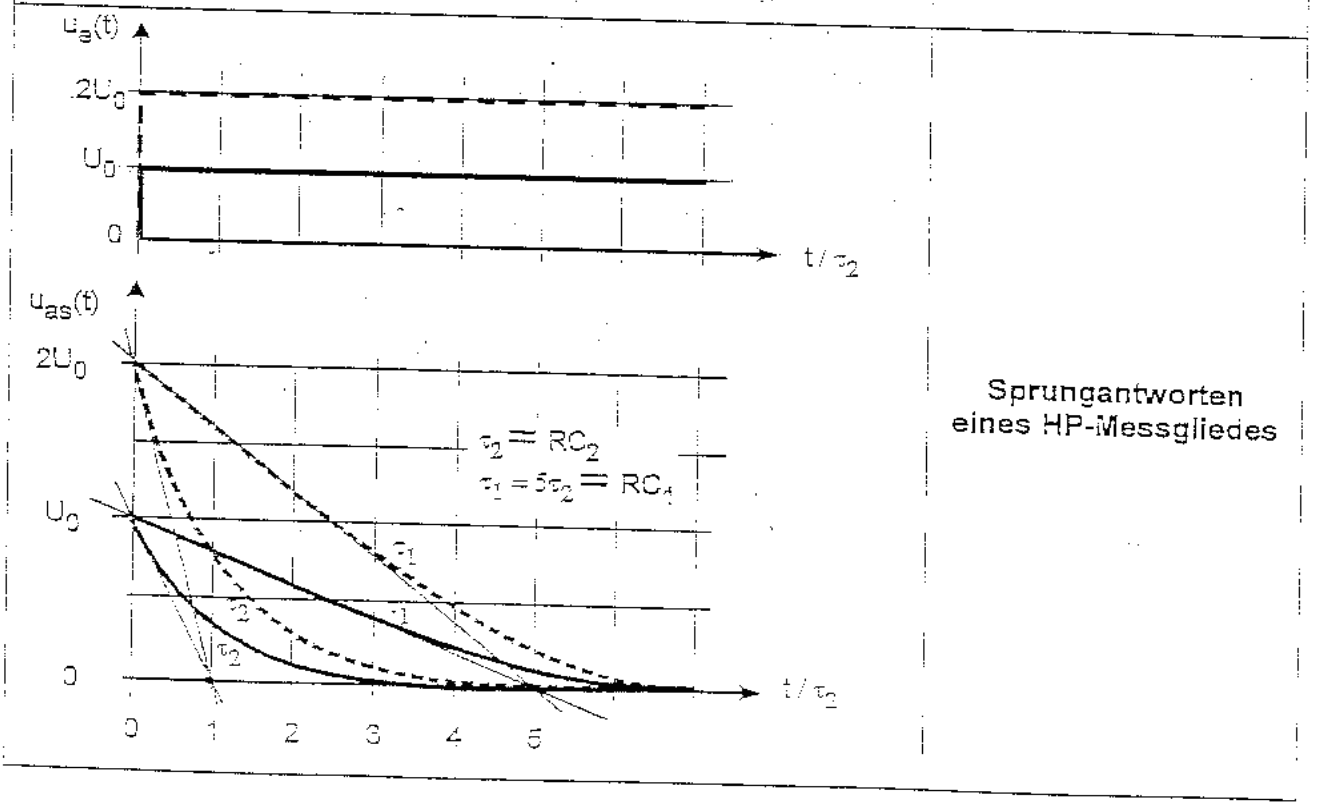
		Messglied Typ A	Messglied Typ B
j)	Lösen der DGL. von f) für Sprung von 0V->U_0	$\boxed{u_{as}(t)=U_0(1-e^{-t/\tau}) \quad mit\ \tau=RC}$ $\boxed{h(t)=u_{as}(t)/U_0=(1-e^{-t/\tau})}$	$\boxed{u_{as}(t)=U_0\,e^{-t/\tau} \quad mit\ \tau=RC}$ $\boxed{h(t)=u_{as}(t)/U_0=e^{-t/\tau})}$
k)		siehe separate Diagramme	siehe separate Diagramme
l)	Betrachtung im Zeitbereich Betr. im Frequenzbereich	=> Tiefpaßverhalten, da G (f<<f_g) ≈ 1; => Integrierer, da G (f >> f_g) ≈ 1/jω	=> Hochpaßverhalten, da G (f>>f_g) ≈ 1; => Differenzierer, da G (f << f_g) ≈ jω


Amplitudengang eines TP-Messgliedes
G(f)
1
0,5
0,1
0,05
0,01
6 Hz
1 Hz
0,1
0,5
1
5
10
50
100
Frequenz in Hz


Phasengang eines TP-Messgliedes
φ(f)
0
-45
-90
0,1
0,5
1
5
10
50
100
Frequenz in Hz

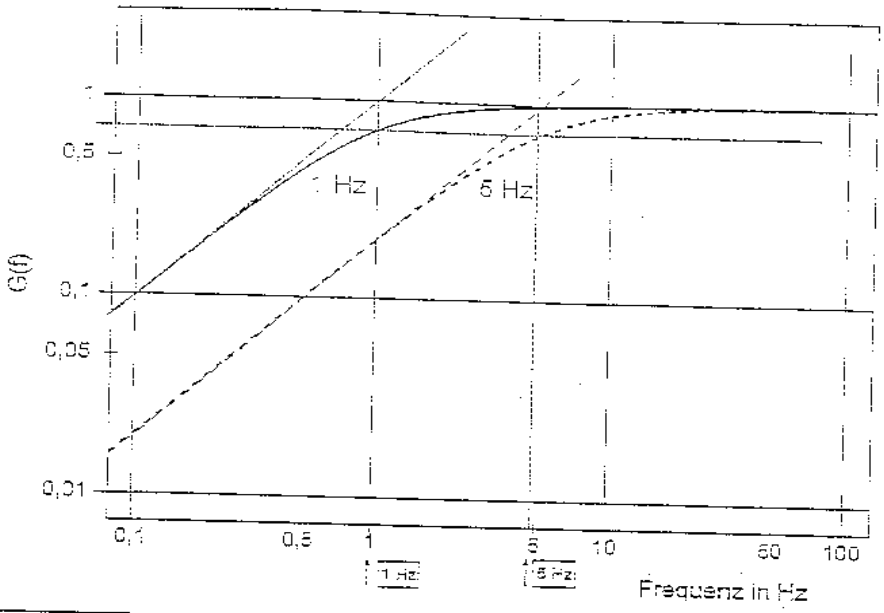


Sprungantworten eines TP-Messgliedes

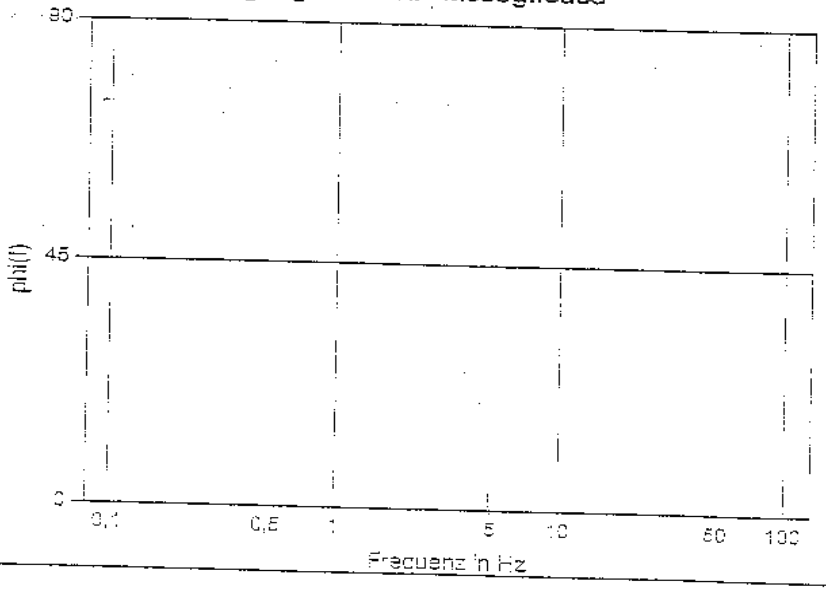


Sprungantworten eines HP-Messgliedes

Amplitudengang eines HP-Messgliedes



Phasengang eines HP-Messgliedes



2.2 komplexer Spannungsteiler

allg. $$\frac{\underline{U}_2}{\underline{U}_1}=\frac{\underline{Z}_2}{\underline{Z}_1+\underline{Z}_2}=\frac{1}{1+\frac{\underline{Z}_1}{\underline{Z}_2}}$$

a) $$\underline{Z}_1=R_1\,;\underline{Z}_2=R_2\|C_2=\frac{R_2}{1+j\omega R_2C_2} \quad\Rightarrow\quad \boxed{\frac{\underline{U}_2}{\underline{U}_1}==\frac{1}{1+\frac{R_1}{R_2}(1+j\omega R_2C_2)}}$$

b) $$\underline{Z}_1=R_1\|C_1=\frac{R_1}{1+j\omega R_1C_1}\,;\,\underline{Z}_2=R_2\|C_2=\frac{R_2}{1+j\omega R_2C_2} \quad\Rightarrow\quad \boxed{\frac{\underline{U}_2}{\underline{U}_1}==\frac{1}{1+\frac{R_1}{R_2}\,\frac{1+j\omega R_2C_2}{1+j\omega R_1C_1}}}$$

c) $$\frac{\underline{U}_2}{\underline{U}_1}==\frac{1}{1+\frac{R_1}{R_2}}\neq g(\omega) \quad\text{für}\quad \frac{1+j\omega R_2C_2}{1+j\omega R_1C_1}=1 \quad\Rightarrow\boxed{R_1C_1=R_2C_2}$$