

3.1

$$R = U / I$$

a) 5312e

$$b) \underline{\overline{F_{abs}}} = \sum \frac{\partial F}{\partial x_i} \Delta x_i = \frac{\partial R}{\partial U} \Delta U + \frac{\partial R}{\partial I} \Delta I = \frac{1}{I} \Delta U - \frac{U}{I^2} \Delta I$$

$$= \frac{-100 \text{ mV}}{700 \text{ mA}} - \frac{8000 \text{ mV}}{(700 \text{ mA})^2} \cdot 15 \text{ mA} = \underline{\underline{-0,1455\%}}$$

$$\underline{\overline{F_{rel}}} = \varepsilon_U - \varepsilon_I = \frac{-100 \text{ mV}}{8000 \text{ mV}} - \frac{15 \text{ mA}}{700 \text{ mA}} = \underline{\underline{-0,034\% \approx -3,4\%}}$$

3.2 → Vorlesung

3.3

$$a) \underline{\overline{F_{abs}}} = \sum \frac{\partial F}{\partial x_i} \Delta x_i = \frac{\partial y}{\partial A} \Delta A + \frac{\partial y}{\partial B} \Delta B = \underline{\underline{\left((1+B) \Delta A + A \Delta B \right) K}}$$

$$\text{mit } \varepsilon_A = \frac{\Delta A}{A}; \varepsilon_B = \frac{\Delta B}{B} \Rightarrow \underline{\underline{\left((1+B) A \varepsilon_A + A B \varepsilon_B \right) K}}$$

$$b) \underline{\overline{F_{rel}}} = \frac{\overline{F_{abs}}}{y} = \frac{(1+B) A \varepsilon_A + A B \varepsilon_B}{A(1+B)} = \underline{\underline{\varepsilon_A + \frac{1}{1+B} \varepsilon_B}}$$

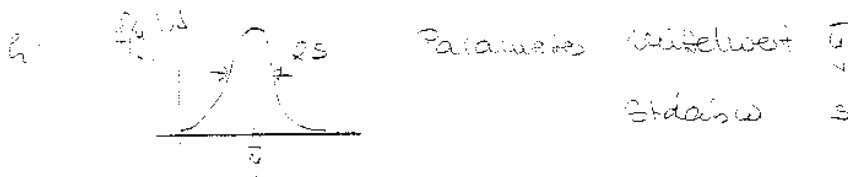
$$c) \underline{\overline{F_{rel}}} = 4,5\%$$

$$d) \text{ Differentiale } \frac{\partial \Delta}{\partial x} \text{ (nicht mehr erlaubt)} \Rightarrow \underline{\overline{F_{rel}}} = \frac{y(A+B) - y(A,B)}{y(A,B)}$$

e), f) ...

$$g) \underline{\overline{F_{abs}}} = \sqrt{\sum \left(\frac{\partial F}{\partial x_i} \right)^2 \Delta x_i^2} = \underline{\underline{\sqrt{(1+B)^2 \varepsilon_A^2 + \varepsilon_B^2}}}$$

$$\underline{\overline{F_{rel}}} = \frac{\overline{F_{abs}}}{y} = \underline{\underline{\sqrt{\varepsilon_A^2 + \frac{1}{1+B} \varepsilon_B^2}}}$$



3.4 Fehlerrechnung allgemein

a) $\bar{F}_{abs} = 2\pi \left(\frac{R}{15} \Delta U + \frac{U}{15} \Delta R - \frac{1}{2} U R S^{-3/2} \Delta S \right)$

b) $\bar{F}_{rel} = \epsilon_u + \epsilon_R - \frac{1}{2} \epsilon_s$ c) d)

e) $\bar{F}_{s abs} = 2\pi \sqrt{\left(\frac{R}{15}\right)^2 s_u^2 + \left(\frac{U}{15}\right)^2 s_R^2 + \left(-\frac{1}{2} U R S^{-3/2}\right)^2 s_s^2}$

$\bar{F}_{s rel} = \sqrt{\epsilon_u^2 + \epsilon_R^2 + \frac{1}{4} \epsilon_s^2}$ f)

3.5 Fehlerrechnung einer Durchflussmessung

a) Empfindlichkeit $E = \frac{\text{Änderung d. Ausgangsgröße}}{\text{d. Eingangsgröße}}$

$\bar{E} = \frac{\partial I}{\partial Q} = \frac{1}{2} a \sqrt{b/Q}$

b) $\bar{F}_{rel} = \epsilon_a + \frac{1}{2} \epsilon_b = |1\%| + |5\%| = \underline{6\% \text{ (max)}}$

3.6 Fehler eines Kapazitätsnormals

a) $\underline{C} = \epsilon_0 \frac{A}{a} = \epsilon_0 \frac{(D/2)^2 \pi}{a} = \frac{\epsilon_0 \pi}{4} \frac{D^2}{a}$

b) $\bar{F}_{abs} = \sum \frac{\partial f}{\partial x_i} \Delta x_i = \frac{\epsilon_0 \pi}{4} (2D \Delta D - a^2 \Delta a)$

$\bar{F}_{rel} = 2 \epsilon_D - \epsilon_a$

c) $\bar{F}_{s abs} = \sqrt{\sum \left(\frac{\partial f}{\partial x_i}\right)^2 s_{x_i}^2} = \frac{\epsilon_0 \pi}{4} \sqrt{(2D)^2 s_D^2 + (a^2)^2 s_a^2}$

$\bar{F}_{s rel} = \sqrt{\sum \left(\frac{\partial f}{\partial x_i}\right)^2 \epsilon_{x_i}^2} = \sqrt{4 \epsilon_D^2 + \epsilon_a^2}$

3.7 Mittelwertauswertung

a) Gauß- oder Normalverteilung $N(\bar{x}; s)$

↪ Mittelwert \bar{x}

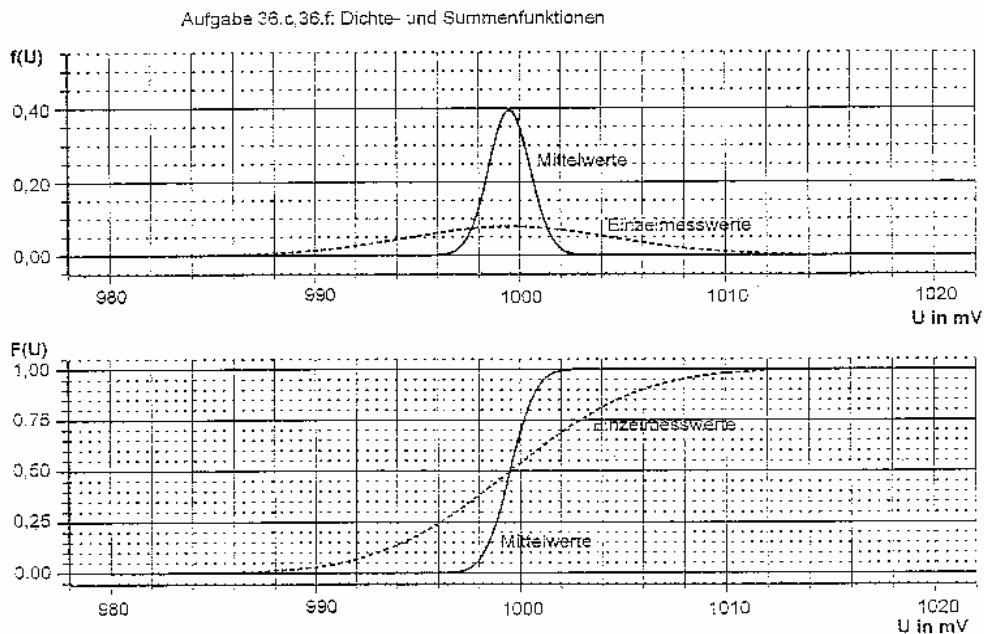
↪ Standardabweichung s

b) ---

c)
$$\bar{U}_{SE} = \frac{\sum U_i}{N} = \frac{995 + \dots + 999}{8} = \underline{\underline{999,5 \text{ mV}}}$$

$$s_{SE} = \sqrt{\frac{1}{N-1} \sum (U_i - \bar{U}_{SE})^2} = \underline{\underline{5,1 \text{ mV}}}$$

d)



e) 68,3%-Intervall $\Delta U = 2 \cdot s_{SE} = 10,1 \text{ mV}$

99,7%- " $\Delta U = 3 \cdot s_{SE} = 17,1 \text{ mV}$

f)
$$s_{FM} = \frac{s_{SE}}{\sqrt{M}} = \frac{5,1}{5} = 1,02 \text{ mV} \quad \mu \text{ sHz} = \mu \text{ s}$$

g) ↪ Mittelwerte streuen sich um Faktor $\frac{1}{\sqrt{M}} = \frac{1}{5}$ weniger als die Einzelmesswerte

h) siehe c)

Aufgabe 3.7 : Dichte- und Summenfunktionen

